ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$, then prove that:

$$\sum \frac{(a+b)(a^2+b^2)}{4c} \geq \sum a^2$$

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$$\sum \frac{(a+b)(a^2+b^2)}{4c} \overset{AM-GM}{\geq} \frac{2ab(a+b)}{4c} = \sum \frac{ab(a+b)}{2c} \geq \sum a^2$$

so it is sufficient to prove that $\sum \frac{ab(a+b)}{c} \geq 2 \cdot \left(\sum a^2\right)$

Grouping the terms from the left – hand side and using the AM-GM inequality, we obtain

$$\left(\frac{a^2b}{c} + \frac{a^2c}{b}\right) + \left(\frac{b^2c}{a} + \frac{b^2a}{c}\right) + \left(\frac{c^2b}{a} + \frac{c^2a}{b}\right) \geq \sum 2a^2$$

which is exactly what we wanted to prove.

Equality holds when $ab = ac \Leftrightarrow b = c$ and when $bc = ab \Leftrightarrow a = c \Rightarrow a = b = c$.