

If $a, b, c > 0$, then prove that

$$\sum \frac{a}{b} - \frac{\sum a^2}{\sum ab} \geq 2$$

Proposed by Neculai Stanciu-Romania

Solution by Cosghun Memmedov-Azerbaijan

$$\begin{aligned} \sum \frac{a}{b} &= \sum \frac{a^2}{ab} \stackrel{\text{Bergstrom}}{\geq} \frac{(\sum a)^2}{\sum ab} = \frac{\sum a^2 + 2\sum ab}{\sum ab} = \\ &= 2 + \frac{\sum a^2}{\sum ab} \Rightarrow \sum \frac{a}{b} - \frac{\sum a^2}{\sum ab} \geq 2 \end{aligned}$$