ROMANIAN MATHEMATICAL MAGAZINE

If $a, \mathbf{b}, \mathbf{c} > 0$, *then prove that* :

$$2\sum_{cyc}\frac{a^3c^2}{a+b} \geq \sum_{cyc} a^2b^2$$

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$$2\sum_{cyc}\frac{a^3c^2}{a+b} - \sum_{cyc} a^2b^2 = 2\sum_{cyc}\frac{a^2c^2(a+b-b)}{a+b} - \sum_{cyc} a^2b^2$$

$$= 2\sum_{cyc} a^2c^2 - 2abc\sum_{cyc}\frac{ca}{a+b} - \sum_{cyc} a^2b^2 \overset{CBS}{\geq}$$

$$\sum_{cyc} a^2b^2 - 2abc.\sqrt{\sum_{cyc} a^2b^2}.\sqrt{\sum_{yc}\frac{1}{(a+b)^2}} \overset{A-G}{\geq}$$

$$\sum_{cyc} a^2b^2 - 2abc.\sqrt{\sum_{cyc} a^2b^2}.\sqrt{\sum_{yc}\frac{c}{4abc}} \overset{?}{\geq} 0 \Leftrightarrow \sqrt{\sum_{cyc} a^2b^2} \overset{?}{\geq} \frac{2abc}{\sqrt{4abc}}.\sqrt{\sum_{cyc} a}$$

$$\Leftrightarrow \sum_{cyc} a^2b^2 \overset{?}{\geq} abc\sum_{cyc} a \rightarrow \text{true} \therefore 2\sum_{cyc}\frac{a^3c^2}{a+b} \geq \sum_{cyc} a^2b^2$$

$$\forall\, a, b, c > 0, '' ='' \ \text{iff } a = b = c \ (\text{QED})$$