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If $a_k > 0 \ (k = 1, 2, \dots, n)$, then

$$\sum_{cyclic} \frac{{a_1}^2}{a_1 + a_2} \ge \frac{1}{2}\sum_{k=1}^{n} a_k \ge \sum_{cyclic} \frac{a_1 {a_2}^2}{{a_1}^2 + {a_2}^2}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$LHS: \frac{{a_1}^2}{a_1 + a_2} + \frac{{a_2}^2}{a_2 + a_3} + \frac{{a_3}^2}{a_3 + a_4} + \ \dots \ + \frac{{a_n}^2}{a_n + a_1} \overset{Bergstrom}{\ge}$$

$$\ge \frac{(a_1 + a_2 + \dots + a_n)^2}{2(a_1 + a_2 + \dots + a_n)} = \frac{1}{2}\sum_{k=1}^{n} a_k$$

$$RHS: \sum_{cyc} \frac{a_1 {a_2}^2}{{a_1}^2 + {a_2}^2} = \sum_{cyc} \frac{a_1}{\left(\frac{a_1}{a_2}\right)^2 + 1} \overset{A-G}{\le} \sum_{cyc} \frac{a_1}{2 \cdot \frac{a_1}{a_2}} = \frac{1}{2}\sum_{cyc} a_k = \frac{1}{2}\sum_{k=1}^{n} a_k$$