ROMANIAN MATHEMATICAL MAGAZINE

$$If\ a_k > 0\ (k = 1, 2, \dots, n) and\ \sum_{k=1}^{n} a_k^2 = n\ \ then\ prove\ that:$$

$$\sum_{k=1}^{n} \frac{1}{n + 1 - ak} \le 1$$

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$$by\ using\ AM - GM:$$

$$1 = \sqrt{\frac{n}{n}} = \sqrt{\frac{a_1^2 + a_2^2 + \cdots .. + a_n^2}{n}} \le \frac{a_1 + a_2 + \cdots . + a_n}{n}$$

$$1 \le \frac{a_1 + a_2 + \cdots . + a_n}{n} \Rightarrow n \le a_1 + a_2 + \cdots . + a_n\ (\mathrm{I})$$

$$but: \sum_{k=1}^{n} a_k^2 = n \Rightarrow \sum_{k=1}^{n} \frac{a_k^2}{n} = 1$$

$$1 = \frac{a_1^2}{n} + \frac{a_2^2}{n} + \cdots + \frac{a_n^2}{n} \overset{BERGSTROM}{\ge} \frac{(a_1 + a_2 + \cdots . + a_n)^2}{n^2}$$

$$(a_1 + a_2 + \cdots . + a_n)^2 \le n^2 \Rightarrow a_1 + a_2 + \cdots . + a_n \le n\ (\Pi)$$

$$from\ (\mathrm{I}), (\Pi):\ a_1 + a_2 + \cdots + a_n = n$$

$$but: \sqrt{\frac{a_1^2 + a_2^2 + \cdots .. + a_n^2}{n}} \le \frac{a_1 + a_2 + \cdots . + a_n}{n}\ holds\ when:$$

$$a_1 = a_2 = \cdots = a_n, so: a_1 = a_2 = \cdots = a_n = \frac{1}{n}$$

$$so: \sum_{k=1}^{n} \frac{1}{n + 1 - a_k} = \frac{n}{n + 1 - \frac{1}{n}} = \frac{n^2}{n^2 + n - 1} \le 1 \Leftrightarrow n \ge 1\ \ (true)$$