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If $a, b, c > 0$ then:

$$3 \sum_{cyc} ab \cdot \sum_{cyc} \frac{a}{b} - 7 \sum_{cyc} ab \geq 2 \sum_{cyc} a^2$$

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Solution by Tapas Das-India

$$\sum \frac{a}{b} = \sum \frac{a^2}{ab} \stackrel{\text{Bergstrom}}{\geq} \frac{(\sum a)^2}{\sum ab} \quad (1)$$

$$\begin{aligned} 3 \sum ab \sum \frac{a}{b} - 7 \sum ab &\stackrel{(1)}{\geq} 3 \sum ab \frac{(\sum a)^2}{\sum ab} - 7 \sum ab = \\ &= 3 \sum a^2 + 6 \sum ab - 7 \sum ab = 3 \sum a^2 - \sum ab = \\ 2 \sum a^2 + \sum a^2 - \sum ab &\geq 2 \sum a^2 \quad (\text{since } \sum a^2 \geq \sum ab) \end{aligned}$$

Equality holds for $a = b = c$.