ROMANIAN MATHEMATICAL MAGAZINE

If $a_k > 0 \ (k = 1, 2, 3 \dots n)$ then :

$$\sum_{cyclic} \frac{a_1^2}{a_1 + a_2} \geq \frac{1}{2} \sum_{k=1}^{n} a_k \geq \sum_{cyclic} \frac{a_1 a_1^2}{a_1^2 + a_2^2}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyclic} \frac{a_1^2}{a_1 + a_2} \geq \frac{1}{2} \sum_{k=1}^{n} a_k \geq \sum_{cyclic} \frac{a_1 a_1^2}{a_1^2 + a_2^2}$$

$$\frac{a_1^2}{a_1 + a_2} + \frac{a_2^2}{a_2 + a_3} + \frac{a_3^2}{a_3 + a_4} + \dots + \frac{a_n^2}{a_n + a_1} \overset{Bergstrom}{\overbrace{\geq}}$$

$$\geq \frac{(a_1 + a_2 + a_3 + \cdots a_n)^2}{2(a_1 + a_2 + a_3 + \cdots a_n)} = \frac{1}{2} \sum_{k=1}^{n} a_k$$

$$\sum_{cyclic} \frac{a_1 a_1^2}{a_1^2 + a_2^2} = \sum_{cyc} \frac{a_1}{\left(\frac{a_1}{a_2}\right)^2 + 1} \overset{A-G}{\overbrace{\leq}} \sum_{cyc} \frac{a_1}{2 \cdot \frac{a_1}{a_2}} = \frac{1}{2} \sum_{cyc} a_2 = \frac{1}{2} \sum_{k=1}^{n} a_k \ (true)$$