

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$ then:

$$\sum \frac{a^2 + 2bc}{(a+b+1)(a+b+c^2)} \leq 1$$

Proposed by Neculai Stanciu-Romania

Solution by Tapas Das-India

$$\begin{aligned} & (a+b+1)(a+b+c^2) = \\ & = \left((\sqrt{a+b})^2 + (1)^2 \right) \left((\sqrt{a+b})^2 + (c)^2 \right) \stackrel{c-s}{\geq} (a+b+c)^2 \quad (1) \\ & \sum \frac{a^2 + 2bc}{(a+b+1)(a+b+c^2)} \stackrel{(1)}{\leq} \sum \frac{a^2 + 2bc}{(a+b+c)^2} = \\ & = \frac{a^2 + b^2 + c^2 + 2bc + 2ca + 2ab}{(a+b+c)^2} = \frac{(a+b+c)^2}{(a+b+c)^2} = 1 \end{aligned}$$

Equality holds for $a = b = c = 1$