Prove that if $a, b, c > 0$, then:

$$\sqrt{\sum (a+b)^2} \geq \left(\sqrt{\sum a^2} + \sqrt{3} \sum a \right) \cdot \frac{1}{2}$$

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Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

By CBS inequality, we have

$$2\sqrt{\sum (a+b)^2} = \sqrt{(1+3)\left(\sum a^2 + \left(\sum a \right)^2 \right)} \geq \sqrt{\sum a^2} + \sqrt{3} \sum a,$$

as desired. Equality holds iff $a = b = c$.