ROMANIAN MATHEMATICAL MAGAZINE

Let $a,b,c \geq 0, ab+bc+ca>0$. Prove that

$$\frac{b+c}{ab+ac+2bc}+\frac{c+a}{bc+ba+2ca}+\frac{a+b}{ca+cb+2ab}\leq\frac{3}{2}\cdot\frac{a+b+c}{ab+bc+ca}$$

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$$2(ab+bc+ca)\sum_{cyc}\frac{b+c}{ab+ac+2bc}=\sum_{cyc}\left(b+c+\frac{a(b+c)^2}{ab+ac+2bc}\right)$$

$$\overset{CBS}{\leq} 2\sum_{cyc}a+\sum_{cyc}a\left(\frac{b^2}{ab+bc}+\frac{c^2}{ac+bc}\right)=2\sum_{cyc}a+\sum_{cyc}\left(\frac{ab}{a+c}+\frac{ca}{a+b}\right)$$

$$=2\sum_{cyc}a+\sum_{cyc}\left(\frac{ca}{c+b}+\frac{ab}{b+c}\right)=2\sum_{cyc}a+\sum_{cyc}a=3(a+b+c),$$

as desired. Equality holds iff $(a=b=c>0)$ and

$(a=0,b,c>0)$ and permutation.