ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC, the following relationship holds :

$$\sum_{cyc}\frac{m_a m_b}{m_a+m_b-m_c}\ge\sum_{cyc}m_a$$

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$$\text{We shall prove that } \forall\,\Delta\,ABC:\sum_{cyc}\frac{ab}{a+b-c}\ge\sum_{cyc}a\to(1)\text{ and }(1)$$

$$\Leftrightarrow\sum_{cyc}\frac{ab}{2(s-c)}\ge 2s\Leftrightarrow\sum_{cyc}\frac{bc}{s(s-a)}\ge 4\Leftrightarrow\sum_{cyc}\sec^2\frac{A}{2}\ge 4\Leftrightarrow\frac{(4R+r)^2+s^2}{s^2}\ge 4$$

$$\Leftrightarrow(4R+r)^2\ge 3s^2\to\text{true via Trucht (Doucet)}\therefore(1)\text{ is true and implementing}$$

$$(1)\text{ on a triangle with sides }\frac{2m_a}{3},\frac{2m_b}{3},\frac{2m_c}{3},\text{ we arrive at :}$$

$$\sum_{cyc}\frac{\frac{4}{9}.m_a m_b}{\frac{2}{3}(m_a+m_b-m_c)}\ge\frac{2}{3}\sum_{cyc}m_a\therefore\sum_{cyc}\frac{m_a m_b}{m_a+m_b-m_c}\ge\sum_{cyc}m_a$$

$$\forall\,\Delta\,ABC,''=''\text{ iif }\Delta\,ABC\text{ is equilateral (QED)}$$