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In ΔABC the following relationship holds:

$$\sum \frac{\cos\frac{A-B}{2}}{12\sin^2\frac{C}{2}} \geq 1$$

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Solution by Tapas Das-India

$$Mollweide's: \frac{\cos\frac{A-B}{2}}{\sin\frac{C}{2}} = \frac{a+b}{c}$$

$$\frac{\cos\frac{A-B}{2}}{12\sin^2\frac{C}{2}} = \frac{1}{12}\cdot\frac{a+b}{c}\cdot\frac{1}{\sin\frac{C}{2}} \overset{AM-GM}{\geq} \frac{2\sqrt{ab}}{12c}\cdot\frac{1}{\sin\frac{C}{2}} = \frac{\sqrt{ab}}{6c}\cdot\frac{1}{\sin\frac{C}{2}}$$

$$\sum \frac{\cos\frac{A-B}{2}}{12\sin^2\frac{C}{2}} \geq \sum \frac{\sqrt{ab}}{6c}\frac{1}{\sin\frac{C}{2}} \overset{AM-GM}{\geq} \frac{3}{6}\sqrt[3]{\prod \csc\frac{C}{2}} = \frac{1}{2}\left(\frac{4R}{r}\right)^{\frac{1}{3}} \overset{Euler}{\geq} \frac{1}{2}(8)^{\frac{1}{3}} = 1$$

Equality holds for an equilateral triangle.