ROMANIAN MATHEMATICAL MAGAZINE

In any ΔABC we have:

$$\frac{a^2}{\sin^2 x}+\frac{b^2}{\cos^2 x}\geq\left(2\sqrt{11r^2+8Rr}-c\right)^2\ \forall x\in R-\left\{\frac{k\pi}{2}\right\}$$

Proposed by Radu Diaconu-Romania

Solution by Tapas Das-India

$$s^2\overset{Gerretsen}{\geq}16Rr-5r^2=8Rr+8Rr-5r^2\overset{Euler}{\geq}$$

$$\geq 8.(2r).r+8Rr-5r^2=16r^2+8Rr-5r^2=11r^2+8Rr\ \ (1)$$

$$\frac{a^2}{\sin^2 x}+\frac{b^2}{\cos^2 x}\overset{Bergstrom}{\geq}\frac{(a+b)^2}{\cos^2 x+\sin^2 x}=(a+b)^2=(2s-c)^2=$$

$$=\left(2\sqrt{s^2}-c\right)^2\overset{(1)}{\geq}\left(2\sqrt{11r^2+8Rr}-c\right)^2$$