PP37383

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If $x, y, z \in \mathbb{R}$ then:

$$\frac{(x^{12}+x^6+1)(y^{24}+y^{12}+1)(z^{36}+z^{18}+1)}{(x^8+1)(y^{16}+1)(z^{24}+1)} \geq x^2y^4z^6$$

Solution by Rovsen Pirguliyev - Azerbaijan.
To prove that

$$x^6 - x^5 + x^3 - x + 1 > 0 \tag{1}$$

for all $x \in \mathbb{R}$.
1) $x < 0, x^6 - x^5 + x^3 - x + 1 \geq 1 > 0$ true.
2) $0 < x < 1, x^6 + (x^3 - x^5) + (1 - x) > 0$ true.
3) $x \geq 1, x^6 - x^5 + x^3 - x + 1 = x^5(x-1) + x(x-1)(x+1) + 1 =$
$= (x-1)(x^5 + x^2 + x) + 1 \geq 1 > 0$ true.
Take in (1) $x \to x^2, y^4, z^6$, we have:

$$x^{12} - x^{10} + x^6 - x^2 + 1 > 0 \Rightarrow \frac{x^{12}+x^6+1}{x^8+1} > x^2 \tag{2}$$

$$y^{24} - y^{20} + y^{12} - y^4 + 1 > 0 \Rightarrow \frac{y^{24}+y^{12}+1}{y^{16}+1} > y^4 \tag{3}$$

$$z^{36} - z^{30} + z^{18} - z^6 + 1 > 0 \Rightarrow \frac{z^{36}+z^{18}+1}{z^{24}+1} > z^6 \tag{4}$$

$$(2) \times (3) \times (4) \Rightarrow LHS > x^2y^4z^6$$

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