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If $0 < a \leq b < 1$ then:
$$\sin\left(\frac{3a+b+2}{4}\right)\sin\left(\frac{a+3b+6}{4}\right) \leq \sin\left(\frac{a+3b+2}{4}\right)\sin\left(\frac{3a+b+6}{4}\right)$$

Solution by Rovsen Pirguliyev - Azerbaijan.
Using the formula
$$\sin x \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y))$$
we have:
$$\frac{1}{2}\cdot\left(\cos\frac{a-b-2}{2} - \cos\frac{4a+4b+8}{4}\right) \leq \frac{1}{2}\cdot\left(\cos\frac{b-a-2}{2} - \cos\frac{4a+4b+8}{4}\right) \text{ or}$$
$$\cos\frac{a-b-2}{2} - \cos\frac{b-a-2}{2} \leq 0$$
Using the formula
$$\cos x - cosy = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$
we have:
$$-2\sin\frac{\frac{a-b-2}{2}+\frac{b-a-2}{2}}{2}\sin\frac{\frac{a-b-2}{2}-\frac{b-a-2}{2}}{2} \leq 0 \text{ or}$$
$$-2\sin(-1)\sin\frac{a-b}{2} \leq 0 \quad \text{or} \quad 2\sin 1\cdot\sin\frac{a-b}{2} \leq 0$$
It is true! Since $\sin 1 > 0$ and $\sin\frac{a-b}{2} \leq 0$ (since $0 < a \leq b < 1$).
Since all inequalities are equivalent the proof is complete. □

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