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In any ABC triangle with the area F the following inequality holds:

$$m_a^2 w_a^2 + m_b^2 w_b^2 + m_c^2 w_c^2 \geq 243 r^3$$

Solution by Rovsen Pirguliyev - Azerbaijan.

It is known that:

$$m_a m_b m_c w_a w_b w_c \geq s^4 r^2 \tag{1}$$

and

$$s^2 \geq 27 r^2 \tag{2}$$

Using (1) and (2), we have:

$$LHS \overset{\text{AM-GM}}{\geq} 3\sqrt[3]{m_a^2 m_b^2 m_c^2 w_a^2 w_b^2 w_c^2} \overset{(1)}{\geq} 3\sqrt[3]{s^8 r^4} \overset{(2)}{\geq} 3\sqrt[3]{3^{12} r^{12}} = 3 \cdot 3^4 r^4 = 243 r^4$$

□

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