PP39989

D.M. BĂTINEŢU - GIURGIU, DANIEL SITARU - ROMANIA

Let be $x, y, z > 0$ and $t \geq 0$, then in ABC triangle with the area F and the other usual notations the following inequality holds:

$$\frac{y+z+2t}{x+t} \cdot a^4 + \frac{z+x+2t}{y+t} \cdot b^4 + \frac{x+y+2t}{z+t} \cdot c^4 \geq 32F^2$$

Solution by Rovsen Pirguliyev - Azerbaijan.

$$LHS = \left(\frac{y+z+2t}{x+t} + 1 - 1\right)a^4 + \left(\frac{z+x+2t}{y+t} + 1 - 1\right)b^4 + \left(\frac{x+y+2t}{z+t} + 1 - 1\right)c^4 =$$

$$= \frac{x+y+z+3t}{x+t}(a^2)^2 + \frac{x+y+z+3t}{y+t}(b^2)^2 + \frac{x+y+z+3t}{z+t}(c^2)^2 - (a^4+b^4+c^4) \geq$$

$$\geq (x+y+z+3t) \cdot \frac{(a^2+b^2+c^2)^2}{x+y+z+3t} - (a^4+b^4+c^4) = 2(a^2b^2+b^2c^2+c^2a^2) \geq 2 \cdot 16F^2 = 32F^2$$

$$(a^2b^2 + b^2c^2 + c^2a^2 \geq 16F^2 \qquad \text{(4.12) Geometric ineq. Bottema)}$$

□

Mathematics Department, National Economic College "Theodor Costescu", Drobeta Turnu - Severin, Romania
Email address: dansitaru63@yahoo.com