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If $x, y, z \in \mathbb{R}_+^* = (0, \infty)$, then in ΔABC triangle the following inequality holds:

$$\frac{x}{\sqrt{yz}} \cdot \frac{a}{h_a} + \frac{y}{\sqrt{zx}} \cdot \frac{b}{h_b} + \frac{z}{\sqrt{xy}} \cdot \frac{c}{h_c} \geq 2\sqrt{3}$$

Solution by Rovsen Pirguliyev - Azerbaijan.
Using AM-GM, we have:

$$(1) \qquad LHS \geq 3\sqrt[3]{\frac{xyz \cdot abc}{\sqrt{yz} \cdot \sqrt{zx} \cdot \sqrt{xy} \cdot h_a h_b h_c}} = 3\sqrt[3]{\frac{abc}{h_a h_b h_c}}$$

Applying the known inequalities

$$(abc)^2 \geq \Big(\frac{4F}{\sqrt{3}}\Big)^3 \text{ and } (h_a h_b h_c)^{\frac{1}{3}} \leq 3^{\frac{1}{4}} \cdot F^{\frac{1}{2}} \quad (6.27)$$

Geometric inequality, Bottema, (1968), we have:

$$(1) \Rightarrow LHS \geq 3\sqrt[3]{\frac{(\frac{4F}{\sqrt{3}})^{\frac{3}{2}}}{3^{\frac{3}{4}} \cdot F^{\frac{3}{2}}}} = 3 \cdot 2 \cdot \sqrt[3]{3^{-\frac{3}{2}}} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$$

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