PP40250

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If $a, b, c > 0$ then:

$$\frac{\sqrt{(a^2+b^2)(a^2+c^2)}}{a^2+bc} + \frac{\sqrt{(b^2+c^2)(b^2+a^2)}}{b^2+ca} + \frac{\sqrt{(c^2+b^2)(c^2+a^2)}}{c^2+ab} \geq 3$$

Solution by Rovsen Pirguliyev - Azerbaijan.

$$(*) \qquad \frac{\sqrt{(a^2+b^2)(a^2+c^2)}}{a^2+bc} + \frac{\sqrt{(b^2+c^2)(b^2+a^2)}}{b^2+ca} + \frac{\sqrt{(c^2+b^2)(c^2+a^2)}}{c^2+ab} \geq 3$$

To prove that

$$(1) \qquad \frac{\sqrt{(a^2+b^2)(a^2+c^2)}}{a^2+bc} \geq 1$$

$$\sqrt{(a^2+b^2)(a^2+c^2)} \geq a^2+bc \Leftrightarrow (a^2+b^2)(a^2+c^2) \geq (a^2+bc)^2 \Leftrightarrow$$
$$\Leftrightarrow a^4+a^2c^2+a^2b^2+b^2c^2 \geq a^4+2a^2bc+b^2c^2 \Leftrightarrow$$
$$\Leftrightarrow a^2c^2-2a^2bc+a^2b^2 \geq 0 \Rightarrow (ac-ab)^2 \geq 0 \text{ or}$$
$$a^2(c-b)^2 \geq 0 \text{ it is true!}$$

Similarly we have:

$$(2) \qquad \frac{\sqrt{(b^2+c^2)(b^2+a^2)}}{b^2+ca} \geq 1$$

$$(3) \qquad \frac{\sqrt{(c^2+b^2)(c^2+a^2)}}{c^2+ab} \geq 1$$

$$(1)+(2)+(3) \Rightarrow LHS \geq 3 \qquad (*)$$

□

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