PP40254

DANIEL SITARU - ROMANIA

If $x, y \geq 0$ then:

$$x^3y^3(x+y)^3 \leq (x^2+y^2)(x^3+y^3)(x^4+y^4)$$

Solution by Rovsen Pirguliyev - Azerbaijan.

To prove that

$$x^3+y^3 \geq xy(x+y) \tag{1}$$

$$(x+y)(x^2-xy+y^2)-xy(x+y) \geq 0 \text{ or } (x+y)(x^2-2xy+y^2) \geq 0$$
$$\text{or } (x+y)(x-y)^2 \geq 0 \text{ it is true (since } x,y \geq 0)$$

Now let's prove that

$$(x^2+y^2)(x^4+y^4) \geq x^2y^2(x+y)^2 \tag{2}$$

Brute force!

$$x^6+x^2y^4+x^4y^2+y^6 \geq x^4y^2+2x^3y^3+x^2y^4 \text{ or}$$
$$x^6-2x^3y^3+y^6 \geq 0 \text{ or } (x^3-y^3)^2 \geq 0\text{, true!}$$

Multiplying $(1);(2) \Rightarrow$

$$\Rightarrow (x^2+y^2)(x^3+y^3)(x^4+y^4) \geq x^3y^3(x+y)^3$$

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Mathematics Department, National Economic College "Theodor Costescu", Drobeta Turnu - Severin, Romania

Email address: dansitaru63@yahoo.com