PP40281

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If $x, y, z \in \mathbb{R}_+^* = (0, \infty)$ and $xyz = 1$ then in any ABC triangle with the area F the following inequality holds:

$$xa^2 + yb^2 + zc^2 \geq 4\sqrt{3}F$$

Solution by Rovsen Pirguliyev - Azerbaijan.
Using AM-GM inequality and $xyz = 1$ we have:

$$xa^2 + yb^2 + zc^2 \geq 3\sqrt[3]{xyz \cdot (abc)^2} = 3\sqrt[3]{(abc)^2} \tag{1}$$

Since $(abc)^2 \geq (\frac{4F}{\sqrt{3}})^3$ (Carlitz, Geometric, ineq. Bottema), then we have:

$$(1) \Rightarrow LHS \geq 3\sqrt[3]{(abc)^2} \geq 3 \cdot \sqrt[3]{\left(\frac{4F}{\sqrt{3}}\right)^3} = 4\sqrt{3}F$$

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