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In acute ΔABC the following relationship holds:

$$\frac{\log_{\sin A}\sin B}{\tan\frac{A}{2}}+\frac{\log_{\sin B}\sin C}{\tan\frac{B}{2}}+\frac{\log_{\sin C}\sin A}{\tan\frac{C}{2}}\geq 3\sqrt{3}$$

Solution by Rovsen Pirguliyev - Azerbaijan.

Applying $AM-GM$ inequality to the left side, we have:

$$LHS\geq 3\sqrt[3]{\frac{\log_{\sin A}\sin B\cdot\log_{\sin B}\sin C\cdot\log_{\sin C}\sin A}{\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}}} \tag{1}$$

since $\log_{\sin A}\sin B\cdot\log_{\sin B}\sin C\cdot\log_{\sin C}\sin A=\dfrac{\ln\sin B}{\ln\sin A}\cdot\dfrac{\ln\sin C}{\ln\sin B}\cdot\dfrac{\ln\sin A}{\ln\sin C}=1$ and using the inequality

$$\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}\leq\frac{1}{9}\sqrt{3}$$

(2.34, Geo-ineq. Bottema, 1968), finally we have:

$$(1)\Rightarrow 3\sqrt[3]{\frac{\log_{\sin A}\sin B\cdot\log_{\sin B}\sin C\cdot\log_{\sin C}\sin A}{\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}}}\geq 3\sqrt[3]{\frac{1}{\frac{\sqrt{3}}{9}}}=3\sqrt{3}$$

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