PP40298

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Find $x, y, z > 0$ such that:

$$\begin{cases} \log_2 x + \log_2 y + \log_2 z = 3 \\ 3^x + 3^y + 3^z = 27 \end{cases}$$

Solution by Rovsen Pirguliyev - Azerbaijan.

$$\log_2 x + \log_2 y + \log_2 z = 3 \tag{1}$$

$$3^x + 3^y + 3^z = 27 \tag{2}$$

From the first equation we have:

$$xyz = 8 \tag{3}$$

Twice applying AM-GM inequality to the second equation, we have:

$$27 = 3^x + 3^y + 3^z \geq 3\sqrt[3]{3^{x+y+z}} \geq 3\sqrt[3]{3^{3\sqrt[3]{xyz}}} \overset{(3)}{=} 27$$

Equality is only possible $x = y = z$, then using $x, y, z > 0$, we have: $(2; 2; 2)$ □

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