PP40300

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Let $ABCD$ be a tetrahedron where:

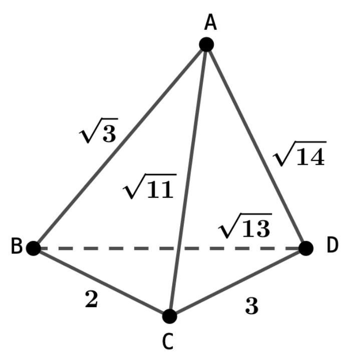
$$AC = \sqrt{11}; CD = 3; AD = \sqrt{14}; AB = \sqrt{3}; BC = 2; BD = \sqrt{13}$$

Prove that:

$$m(\angle(AB, CD)) >= 90^\circ$$

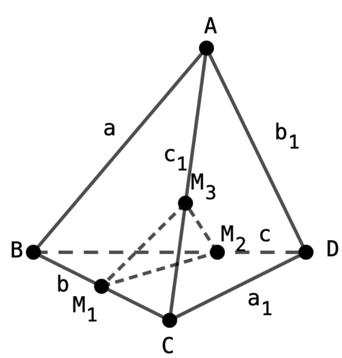
Solution by Rovsen Pirguliyev - Azerbaijan.

$$\text{Let } a = \sqrt{3}, a_1 = 3, b = 2, b_1 = \sqrt{14}, c = \sqrt{13}, c_1 = \sqrt{11}$$



To prove that

$$\cos\varphi = \frac{-b^2 - b_1^2 + c^2 + c_1^2}{2aa_1} \tag{1}$$



prove: Let M_1M_2, M_1M_3 middle lines of the triangles $\Delta BCD, \Delta ABC$.
We have:

$$M_3M_2^2 = \frac{a_1^2}{4} + \frac{a^2}{4} - 2 \cdot \frac{a_1}{2} \cdot \frac{a}{2} \cos\varphi \tag{2}$$

It is known that

$$M_3M_2^2 = \frac{1}{4}(a^2 + a_1^2 + b^2 + b_1^2 - c^2 - c_1^2) \tag{3}$$

Using (3) in (2) $\Rightarrow$

$$\cos\varphi = \frac{-b^2 - b_1^2 + c^2 + c_1^2}{2aa_1}$$

Then we have:

$$\cos\varphi = \frac{-2^2 - \sqrt{14}^2 + \sqrt{13}^2 + \sqrt{11}^2}{2 \cdot 3 \cdot \sqrt{3}} = -\frac{2}{3\sqrt{3}} < 0 \Rightarrow \varphi > 90^\circ$$

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