PP40318

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Prove that if $a, b, c \in (2, \infty)$ then:

$$\sum \ln(a-1)\ln(a+1) < \ln(a^{\ln a} \cdot b^{\ln b} \cdot c^{\ln c})$$

Solution by Rovsen Pirguliyev - Azerbaijan.
Applying the inequality $ab \leq \frac{(a+b)^2}{4}$ (AM-GM $\rightarrow 2\sqrt{ab} \leq a+b$), we have:
(1)

$$\ln(a-1)\ln(a+1) < \frac{(\ln(a-1)+\ln(a+1))^2}{4} = \frac{(\ln(a^2-1))^2}{4} < \frac{(\ln a^2)^2}{4} = \ln^2 a$$

Then we have:

$$\sum \ln(a-1)\ln(a+1) < \ln^2 a + \ln^2 b + \ln^2 c = \ln a \cdot \ln a + \ln b \cdot \ln b + \ln c \cdot \ln c =$$
$$= \ln(a^{\ln a}) + \ln(b^{\ln b}) + \ln(c^{\ln c}) = \ln(a^{\ln a} \cdot b^{\ln b} \cdot c^{\ln c})$$

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