PP40423

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Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that:

$$1 + f(x+y) \leq f(x) + f(y) \leq x + y + 2; \quad \forall x, y \in \mathbb{R}$$

Solution by Rovsen Pirguliyev - Azerbaijan.

Take $x = y = 0 \Rightarrow 1 + f(0) \leq 2f(0) \leq 2$, hence

$$f(0) \leq 1, f(0) \geq 1 \Rightarrow f(0) = 1 \tag{1}$$

Take $x = -y, (y = -x) \Rightarrow 1 + f(0) \leq f(x) + f(-x) \leq 2 \Rightarrow$

$\Rightarrow f(x) + f(-x) \geq 2$ and $f(x) + f(-x) \leq 2 \Rightarrow$

$$\Rightarrow f(x) + f(-x) = 2 \tag{2}$$

Take $y = x$ in $f(x) + f(y) \leq x + y + 2$, we have:

$$2f(x) \leq 2x + 2 \Rightarrow f(x) \leq x + 1 \tag{3}$$

in (2) $\Rightarrow f(x) = 2 - f(-x)$, using in (3) $\Rightarrow$

$$2 - f(-x) \leq x + 1 \Rightarrow f(-x) \geq -x + 1 \text{ or } f(t) \geq t + 1 \tag{4}$$

further we have: $(3);(4) \Rightarrow f(x) = x + 1$,

This function satisfies all the inequalities of our problem.

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