# PP40442

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If $x, y, z \geq 0$ then in $\Delta ABC$ the following relationship holds:

$$\frac{e^x + y + 1}{z + 1} \cdot ab + \frac{e^y + z + 1}{x + 1} \cdot bc + \frac{e^z + x + 1}{y + 1} \cdot ca \geq 8\sqrt{3} \cdot F$$

*Solution by Rovsen Pirguliyev - Azerbaijan.*
From the course of math analysis we know that

$$e^x \geq x + 1 \text{ for all } x \geq 0 \tag{1}$$

Using (1) and AM-GM, we have:

$$LHS \geq 3\sqrt[3]{\frac{e^x + y + 1}{y + 1} \cdot \frac{e^y + z + 1}{z + 1} \cdot \frac{e^z + x + 1}{x + 1} \cdot (abc)^2} \overset{\text{using (1)}}{\geq}$$

$$\geq 3\sqrt[3]{\left(1 + \frac{x + 1}{y + 1}\right)\left(1 + \frac{y + 1}{z + 1}\right)\left(1 + \frac{z + 1}{x + 1}\right)(abc)^2} \tag{2}$$

Applying AM-GM inequality for

$$\left(1 + \frac{x + 1}{y + 1}\right)\left(1 + \frac{y + 1}{z + 1}\right)\left(1 + \frac{z + 1}{x + 1}\right) \geq 2\sqrt{\frac{x + 1}{y + 1}} \cdot 2\sqrt{\frac{y + 1}{z + 1}} \cdot 2\sqrt{\frac{z + 1}{x + 1}} = 8 \tag{3}$$

We have:

$$(2) \overset{(3)}{\Rightarrow} 3\sqrt[3]{8(abc)^2} \geq 6\sqrt[3]{(abc)^2} \geq 6 \cdot \sqrt[3]{\left(\frac{4F}{\sqrt{3}}\right)^3} =$$

$$= \frac{24F}{\sqrt{3}} = 8\sqrt{3}F$$

at the last stage, I have applied the inequality $(abc)^2 \geq (\frac{4F}{\sqrt{3}})^3$ (Carlitz, Geometric inequality, O. Bottema, 1968)

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