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If $m, n, p, x, y, z > 0$ then in ΔABC holds:

$$(mx + ny) \cdot \frac{a^2}{z} + (my + nz) \cdot \frac{b^2}{x} + (mz + n) \cdot \frac{c^2}{y} \geq 8\sqrt{3mn} \cdot F$$

Solution by Rovsen Pirguliyev - Azerbaijan.
Using the inequality AM - GM

$$(x + y \geq 2\sqrt{xy}, x + y + z \geq 3\sqrt[3]{xyz}) \text{ and } (abc)^2 \geq \left(\frac{4F}{\sqrt{3}}\right)^3$$

(L. Carlitz, 4.14 Geometric inequalities, Bottema)
we have:

$$LHS \geq 2\sqrt{mnxy} \cdot \frac{a^2}{z} + 2\sqrt{mnyz} \cdot \frac{b^2}{x} + 2\sqrt{mnzx} \cdot \frac{c^2}{y} \geq$$

$$\geq 3\sqrt[3]{8\sqrt{(mn)^3} \cdot a^2b^2c^2} \geq 6\sqrt{mn} \cdot \frac{4F}{\sqrt{3}} = 8\sqrt{3mn}F$$

□

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