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MIHÁLY BENCZE - ROMANIA

If $a, b, c > 0$ then:

$$\sum_{cyc} \frac{1}{a^3 + b^3 + 4} \leq \frac{1}{6} \sum_{cyc} \frac{1}{\sqrt{ab}}$$

Solution by Daniel Sitaru.

$$\sum_{cyc} \frac{1}{a^3 + b^3 + 4} = \sum_{cyc} \frac{1}{a^3 + b^3 + 1 + 1 + 1 + 1} \leq$$

$$\overset{\text{AM-GM}}{\leq} \sum_{cyc} \frac{1}{6\sqrt[6]{a^3 b^3 \cdot 1 \cdot 1 \cdot 1 \cdot 1}} = \frac{1}{6} \sum_{cyc} \frac{1}{\sqrt[6]{(ab)^3}} =$$

$$= \frac{1}{6} \sum_{cyc} \frac{1}{\sqrt{ab}}$$

Equality holds for $a = b = c = 1$. □

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