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MIHÁLY BENCZE- ROMANIA

If $a, b > 0$ then:

$$\left(\frac{b^3}{a^2}+3a\right)^2+\left(\frac{a^3}{b^2}+3b\right)^2 \geq 16(a^2+b^2)$$

Solution by Daniel Sitaru.

$$\left(\frac{b^3}{a^2}+3a\right)^2+\left(\frac{a^3}{b^2}+3b\right)^2 \geq 16(a^2+b^2)$$

$$\frac{b^6}{a^4}+9a^2+\frac{6b^3}{a}+\frac{a^6}{b^4}+9b^2+\frac{6a^3}{b} \geq 16a^2+16b^2$$

$$\frac{b^6}{a^4}+\frac{6a^3}{b}+\frac{a^6}{b^4}+\frac{6b^3}{a} \geq 7a^2+7b^2 \text{(to prove)}$$

$$\frac{b^6}{a^4}+\frac{6a^3}{b} \geq 7\cdot\sqrt[7]{\frac{b^6}{a^4}\cdot\left(\frac{a^3}{b}\right)^6} = 7\sqrt[7]{\frac{a^{18}}{a^4}} = 7a^2 \tag{1}$$

$$\frac{a^6}{b^4}+\frac{6b^3}{a} \geq 7\cdot\sqrt[7]{\frac{a^6}{b^4}\cdot\left(\frac{b^3}{a}\right)^6} = 7\sqrt[7]{\frac{b^{18}}{b^4}} = 7b^2 \tag{2}$$

By adding $(1);(2)$:

$$\frac{b^6}{a^4}+\frac{6a^3}{b}+\frac{a^6}{b^4}+\frac{6b^3}{a} \geq 7a^2+7b^2$$

Equality holds for $a = b$. □

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