PP42153

MIHÁLY BENCZE - ROMANIA

If $a_k, b_k > 0 \quad (k \in \overline{1,n})$ then:

$$\sum_{k=1}^{n} \frac{1}{\frac{1}{a_k} + \frac{1}{b_k}} \leq \frac{1}{\frac{1}{\sum_{k=1}^{n} a_k} + \frac{1}{\sum_{k=1}^{n} b_k}}$$

Solution by Daniel Sitaru.

$$\sum_{k=1}^{n} \frac{1}{\frac{1}{a_k} + \frac{1}{b_k}} = \sum_{k=1}^{n} \frac{a_k b_k}{a_k + b_k} = \sum_{k=1}^{n} \Big(\frac{a_k b_k}{a_k + b_k} - a_k + a_k \Big) =$$

$$= \sum_{k=1}^{n} \Big(\frac{a_k b_k}{a_k + b_k} - a_k \Big) + \sum_{k=1}^{n} a_k =$$

$$= \sum_{k=1}^{n} \frac{a_k b_k - a_k^2 - a_k b_k}{a_k + b_k} + \sum_{k=1}^{n} a_k =$$

$$= \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} \frac{a_k^2}{a_k + b_k} \overset{\text{CBS}}{\leq}$$

$$\leq \sum_{k=1}^{n} a_k - \frac{(a_1 + a_2 + \ldots + a_n)^2}{(a_1 + b_1) + (a_2 + b_2) + \ldots + (a_n + b_n)} =$$

$$= \sum_{k=1}^{n} a_k - \frac{\Big(\sum_{k=1}^{n} a_k \Big)^2}{\sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k} =$$

$$= \frac{\Big(\sum_{k=1}^{n} a_k \Big)^2 + \Big(\sum_{k=1}^{n} a_k \Big)\Big(\sum_{k=1}^{n} b_k \Big) - \Big(\sum_{k=1}^{n} a_k \Big)^2}{\sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k} =$$

$$= \frac{\Big(\sum_{k=1}^{n} a_k \Big)\Big(\sum_{k=1}^{n} b_k \Big)}{\sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k} = \frac{1}{\frac{1}{\sum_{k=1}^{n} a_k} + \frac{1}{\sum_{k=1}^{n} b_k}}$$

□

Mathematics Department, National Economic College "Theodor Costescu", Drobeta Turnu - Severin, Romania

Email address: dansitaru63@yahoo.com