PP43022

ROVSEN PIRGULIYEV - AZERBAIJAN

Prove without softs:

$$\arccos\frac{1}{\pi^2}-\arccos\frac{1}{e^2}>\frac{\pi}{2020}(e^{2020}-\pi^{2020})$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\text{Let be } f:(1,\infty)\to\mathbb{R}, f(x)=\arccos\frac{1}{x^2}+\frac{\pi}{2020}\cdot x^{2020}$$

$$f'(x)=\frac{-\left(\frac{1}{x^2}\right)'}{\sqrt{1-\frac{1}{x^4}}}+\frac{\pi}{2020}\cdot 2020^{2019}$$

$$f'(x)=\frac{2}{x^3\sqrt{1-\frac{1}{x^4}}}+\pi x^{2019}>0;(\forall)x>1$$

$$f \text{ increasing}; \pi>e\Rightarrow f(\pi)>f(e)$$

$$\arccos\frac{1}{\pi^2}+\frac{\pi}{2020}\cdot\pi^{2020}>\arccos\frac{1}{e^2}+\frac{\pi}{2020}\cdot e^{2020}$$

$$\arccos\frac{1}{\pi^2}-\arccos\frac{1}{e^2}>\frac{\pi}{2020}(e^{2020}-\pi^{2020})$$

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