PP43098

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc}\frac{m_a^2}{b^2+c^2-a^2}\geq\frac{9}{4}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\sum_{cyc}\frac{m_a^2}{b^2+c^2-a^2}=\sum_{cyc}\frac{\frac{2(b^2+c^2)-a^2}{4}}{b^2+c^2-a^2}=$$

$$=\frac{1}{4}\sum_{cyc}\frac{b^2+c^2-a^2+b^2+c^2}{b^2+c^2-a^2}=$$

$$=\frac{1}{4}\sum_{cyc}\frac{b^2+c^2-a^2}{b^2+c^2-a^2}+\frac{1}{4}\sum_{cyc}\frac{b^2+c^2}{b^2+c^2-a^2}=$$

$$=\frac{3}{4}+\frac{1}{4}\sum_{cyc}\frac{1}{1-\frac{a^2}{b^2+c^2}}\overset{\text{BERGSTRÖM}}{\geq}$$

$$\geq\frac{3}{4}+\frac{1}{4}\cdot\frac{9}{3-\sum_{cyc}\frac{a^2}{b^2+c^2}}\overset{\text{NESBIT}}{\geq}$$

$$\geq\frac{3}{4}+\frac{1}{4}\cdot\frac{9}{3-\frac{3}{2}}=\frac{3}{4}+\frac{9}{12-6}=\frac{3}{4}+\frac{3}{2}=\frac{9}{4}$$

Equality holds for $a=b=c$. □

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