

PP43120

MIHÁLY BENCZE - ROMANIA

Prove that:

$$\int_a^b \frac{(x+1)(x+2)}{x(x^3+x^2+4)} \leq \ln\left(\frac{b}{a}\right); 0 < a \leq b$$

Solution by Daniel Sitaru, Claudia Nănuți.

First we prove that:

$$(1) \quad \begin{aligned} \frac{(x+1)(x+2)}{x(x^3+x^2+4)} &\leq \frac{1}{x}; x > 0 \\ (x+1)(x+2) &\leq x^3+x^2+4 \\ x^2+3x+2 &\leq x^3+x^2+4 \\ x^3-3x+2 &\geq 0 \\ x^3-x-2x+2 &\geq 0 \\ x(x-1)(x+1)-2(x-1) &\geq 0 \\ (x-1)^2(x+2) &\geq 0 \quad (\text{True}) \end{aligned}$$

By integrating (1):

$$\int_a^b \frac{(x+1)(x+2)}{x(x^3+x^2+4)} dx \leq \int_a^b \frac{1}{x} dx = \ln\left(\frac{b}{a}\right)$$

Equality holds for $a = b$.

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