PP43121

MIHÁLY BENCZE - ROMANIA

Prove that:

$$\prod_{k=1}^{n}\Big(\frac{k^2}{2}+\frac{2}{k+1}\Big)\geq\frac{(n+2)!}{2^{n+1}}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$P(n):\prod_{k=1}^{n}\Big(\frac{k^2}{2}+\frac{2}{k+1}\Big)\geq\frac{(n+2)!}{2^{n+1}}$$

For: $n=1$

$$\frac{1^2}{2}+\frac{2}{1+1}\geq\frac{(1+2)!}{2^{1+1}}\Leftrightarrow\frac{1}{2}+1\geq\frac{6}{4}\Leftrightarrow\frac{3}{2}=\frac{3}{2}$$

By mathematical induction suppose $P(n)$ true.
Remains to prove:

$$P(n+1):\prod_{k=1}^{n+1}\Big(\frac{k^2}{2}+\frac{2}{k+1}\Big)\geq\frac{(n+3)!}{2^{n+2}}$$

$$\prod_{k=1}^{n+1}\Big(\frac{k^2}{2}+\frac{2}{k+1}\Big)=\Big(\frac{(n+1)^2}{2}+\frac{2}{n+2}\Big)\cdot\prod_{k=1}^{n}\Big(\frac{k^2}{2}+\frac{2}{k+1}\Big)\geq$$

$$\overset{P(n)}{\geq}\Big(\frac{(n+1)^2}{2}+\frac{2}{n+2}\Big)\cdot\frac{(n+2)!}{2^{n+1}}\geq\frac{(n+3)!}{2^{n+2}}$$

$$\frac{(n+1)^2}{2}+\frac{2}{n+2}\geq\frac{n+3}{2}$$

$$(n+1)^2(n+2)+4\geq(n+2)(n+3)$$

$$n^3+2n^2+2n^2+4n+n+2+4\geq n^2+5n+6$$

$$n^3+4n^2-n^2\geq 0$$

$$n^3(n+4)\geq 0$$

$$P(n)\rightarrow P(n+1)$$

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