PP43411

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc} \frac{r_a}{r_b} \geq 1 + \frac{2(4R+r)^2}{3s^2} \geq 3$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\sum_{cyc} \frac{r_a}{r_b} = \sum_{cyc} \frac{r_a^2}{r_a r_b} \overset{\text{BERGSTROM}}{\geq} \frac{(r_a+r_b+r_c)^2}{r_a r_b + r_b r_c + r_c r_a} =$$

$$= \frac{(4R+r)^2}{s^2} \geq 1 + \frac{2(4R+r)^2}{3s^2} \Leftrightarrow$$

$$\Leftrightarrow \frac{(4R+r)^2}{s^2} - \frac{2(4R+r)^2}{3s^2} \geq 1$$

$$3(4R+r)^2 - 2(4R+r)^2 \geq 3s^2$$

$$(4R+r)^2 \geq 3s^2$$

$$s\sqrt{3} \leq 4R + r \text{ (DOUCET)}$$

$$1 + \frac{2(4R+r)^2}{3s^2} \geq 3$$

$$\frac{2(4R+r)^2}{3s^2} \geq 2$$

$$(4R+r)^2 \geq 3s^2$$

$$s\sqrt{3} \leq 4R + r \text{ (DOUCET)}$$

□

Mathematics Department, National Economic College "Theodor Costescu", Drobeta Turnu - Severin, Romania

Email address: dansitaru63@yahoo.com