PP43492

MIHÁLY BENCZE - ROMANIA

In all acute triangles ABC holds:

$$\cos A \cos B \cos C \leq \frac{r^2}{2R^2}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

It is known that:

$$2 + 2\prod_{cyc} \cos A = \frac{s^2 - r^2 - 4Rr}{2R^2}$$

$$2\prod_{cyc} \cos A = \frac{s^2 - r^2 - 4Rr}{2R^2} - 2$$

$$\prod_{cyc} \cos A = \frac{s^2 - r^2 - 4Rr - 4R^2}{4R^2}$$

Remains to prove:

$$\frac{s^2 - r^2 - 4Rr - 4R^2}{4R^2} \leq \frac{r^2}{2R^2}$$

$$s^2 - r^2 - 4Rr - 4R^2 \leq 2r^2$$

$$s^2 \leq 4R^2 + 4Rr + 3r^2$$

which it is Gerretsen's inequality

□

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