

**PP43553**

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In all triangles  $ABC$  holds:

$$\sum_{cyc} \left(\frac{h_a}{w_a}\right)^2 - 1 = \frac{s^2 + r^2 + 2Rr}{4R^2}$$

*Solution by Daniel Sitaru, Claudia Nănuți.*

$$\begin{aligned} \sum_{cyc} \left(\frac{h_a}{w_a}\right)^2 - 1 &= \sum_{cyc} \cos^2\left(\frac{B-C}{2}\right) - 1 = \\ &= \sum_{cyc} \frac{1 + \cos(B-C)}{2} - 1 = \frac{3}{2} + \frac{1}{2} \sum_{cyc} \cos(B-C) - 1 = \\ &= \frac{1}{2} + \frac{1}{2} \cdot \left(\frac{s^2 + r^2 + 2Rr}{2R^2} - 1\right) = \\ &= \frac{1}{2} + \frac{s^2 + r^2 + 2Rr}{4R^2} - \frac{1}{2} = \\ &= \frac{s^2 + r^2 + 2Rr}{4R^2} \end{aligned}$$

Equality holds for  $a = b = c$ .

□

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