PP43649

MIHÁLY BENCZE - ROMANIA

Compute:

$$\int_1^2 \frac{(x^2-1)^n(x^2+1)}{x^{n+2}}dx; n \in \mathbb{N}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\Omega = \int_1^2 \frac{(x^2-1)^n(x^2+1)}{x^{n+2}}dx = \int_1^2 \frac{(x^2-1)(x^2+1)}{x^n \cdot x^2}dx =$$

$$= \int_1^2 \left(x - \frac{1}{x}\right)^n \left(1 + \frac{1}{x^2}\right)dx$$

Denote: $y = x - \dfrac{1}{x}; dy = \left(1 + \dfrac{1}{x^2}\right)dx$

If $x = 1 \Rightarrow y = 0$

If $x = 2 \Rightarrow y = \dfrac{3}{2}$

$$\Omega = \int_0^{\frac{3}{2}} y^n dy = \frac{y^{n+1}}{n+1}\Big|_0^{\frac{3}{2}} =$$

$$= \frac{1}{n+1} \cdot \left(\frac{3}{2}\right)^{n+1}$$

$\square$

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