PP44056

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Prove without softs:

$$\arcsin\frac{1}{e}-\arcsin\frac{1}{\pi}<\left(1-\frac{e}{\pi}\right)\cdot\frac{1}{\sqrt{e^2-1}}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\text{Let be } f:\left[\frac{1}{\pi},\frac{1}{e}\right]\to\mathbb{R}; f(x)=\arcsin x$$

By Lagrange's theorem:

$$(\exists)c\in\left(\frac{1}{\pi},\frac{1}{e}\right)\text{ such that: } f\left(\frac{1}{e}\right)-f\left(\frac{1}{\pi}\right)=f'(c)\left(\frac{1}{e}-\frac{1}{\pi}\right)$$

$$\frac{1}{\pi}<c<\frac{1}{e}\Rightarrow\frac{1}{\pi^2}<c^2<\frac{1}{e^2}$$

$$-\frac{1}{e^2}<-c^2<-\frac{1}{\pi^2}\Rightarrow 1-\frac{1}{e^2}<1-c^2<1-\frac{1}{\pi^2}$$

$$\sqrt{1-\frac{1}{e^2}}<\sqrt{1-c^2}<\sqrt{1-\frac{1}{\pi^2}}$$

$$\frac{1}{\sqrt{1-\frac{1}{\pi^2}}}<\frac{1}{\sqrt{1-c^2}}<\frac{1}{\sqrt{1-\frac{1}{e^2}}}$$

$$\left(\frac{1}{e}-\frac{1}{\pi}\right)\cdot\frac{1}{\sqrt{1-c^2}}<\left(\frac{1}{e}-\frac{1}{\pi}\right)\cdot\frac{1}{\sqrt{1-\frac{1}{e^2}}}$$

$$f\left(\frac{1}{e}\right)-f\left(\frac{1}{\pi}\right)<\frac{\pi-e}{\pi e}\cdot\frac{e}{\sqrt{e^2-1}}$$

$$\arcsin\frac{1}{e}-\arcsin\frac{1}{\pi}<\frac{\pi-e}{\pi}\cdot\frac{1}{\sqrt{e^2-1}}=\left(1-\frac{e}{\pi}\right)\cdot\frac{1}{\sqrt{e^2-1}}$$

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