PP44155

ROVSEN PIRGULIYEV - AZERBAIJAN

In all triangles ABC holds:

$$a^3 + b^3 + c^3 \geq 72\sqrt{3}r^3$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$a^3 + b^3 + c^3 \overset{\text{AM-GM}}{\geq} 3\sqrt[3]{(abc)^3} =$$
$$= 3abc = 3 \cdot 4RF \overset{\text{EULER}}{\geq} 12 \cdot 2r \cdot F =$$
$$= 24r \cdot rs \overset{\text{MITRINOVIC}}{\geq} 24r^2 \cdot 3\sqrt{3}r =$$
$$= 72\sqrt{3}r^3$$

Equality holds for $a = b = c$. □

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