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If $a, b, c > 0$ then:

$$\sum_{cyc} \frac{a+b}{a^2-ab+b^2} \leq 2\Big(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\Big)$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\sum_{cyc} \frac{a+b}{a^2-ab+b^2} \overset{\text{AM-GM}}{\leq} \sum_{cyc} \frac{a+b}{2\sqrt{a^2b^2}-ab} =$$

$$= \sum_{cyc} \frac{a+b}{2ab-ab} = \sum_{cyc} \frac{a+b}{ab} =$$

$$= \sum_{cyc} \frac{a}{ab} + \sum_{cyc} \frac{b}{ab} = \sum_{cyc} \frac{1}{b} + \sum_{cyc} \frac{1}{a} =$$

$$= 2\sum_{cyc} \frac{1}{a} = 2\Big(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\Big)$$

Equality holds for $a = b = c$. □

Mathematics Department, National Economic College "Theodor Costescu", Drobeta Turnu - Severin, Romania

Email address: dansitaru63@yahoo.com