PP44234

MIHÁLY BENCZE - ROMANIA

Compute:

$$\lim_{n\to\infty}\left(\frac{1}{2}-n^2\int_0^1 x^3(1-x^2)^n dx\right)$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\int_0^1 x^3(1-x^2)^n dx=\int_0^{\frac{\pi}{2}}\sin^3 y\cos^{2n} y\cdot\cos y dy=$$

$$x=\sin y\Rightarrow dx=\cos y dy$$

$$=\int_0^{\frac{\pi}{2}}\sin y\cdot\sin^2 y\cdot\cos^{2n+1} y dy=$$

$$=\int_0^{\frac{\pi}{2}}\sin y(1-\cos^2 y)\cos^{2n+1} y dy=$$

$$=\int_0^{\frac{\pi}{2}}\sin y\cos^{2n+1} y dy-\int_0^{\frac{\pi}{2}}\sin y\cos^{2n+3} y dy=$$

$$=-\frac{\cos^{2n+2} y}{2n+2}\Big|_0^{\frac{\pi}{2}}+\frac{\cos^{2n+4} y}{2n+4}\Big|_0^{\frac{\pi}{2}}=$$

$$=\frac{1}{2n+2}-\frac{1}{2n+4}=\frac{2n+4-2n-2}{(2n+2)(2n+4)}=\frac{1}{2(n+1)(n+2)}$$

$$\lim_{n\to\infty}\left(\frac{1}{2}-n^2\int_0^1 x^3(1-x^2)^n dx\right)=$$

$$=\lim_{n\to\infty}\left(\frac{1}{2}-\frac{n^2}{2(n+1)(n+2)}\right)=\frac{1}{2}-\frac{1}{2}=0$$

□

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