PP44328

MIHÁLY BENCZE - ROMANIA

If $a, b, c > 0$ then:

$$\left(\sum_{cyc} ab\right) \cdot \left(\sum_{cyc} \frac{a^2}{b}\right) \geq \left(\sum_{cyc} a\right) \cdot \left(\sum_{cyc} a^2\right)$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$(ab + bc + ca)\left(\frac{a^2}{b} + \frac{b^2}{c} + \frac{c^2}{a}\right) \geq (a + b + c)(a^2 + b^2 + c^2)$$

$$a^3 + \frac{ab^3}{c} + bc^2 + a^2c + b^3 + \frac{bc^3}{a} + \frac{a^3c}{b} + ab^2 + c^3 \geq$$
$$\geq a^3 + ab^2 + ac^2 + a^2b + b^3 + bc^2 + a^2c + b^2c + c^3$$

$$\frac{ab^3}{c} + \frac{bc^3}{a} + \frac{ca^3}{b} \geq ac^2 + a^2b + b^2c$$

(1) $$a^2b^4 + b^2c^4 + c^2a^4 \geq abc(a^2b + b^2c + c^2a)$$

(2) $$\frac{a^2b^4 + b^2c^4}{2} \geq \sqrt{a^2b^4b^2c^4} = ab^3c^2$$

(3) $$\frac{b^2c^4 + c^2a^4}{2} \geq \sqrt{b^2c^4c^2a^4} = bc^3a^2$$

(4) $$\frac{c^2a^4 + a^2b^4}{2} \geq \sqrt{c^2a^4a^2b^4} = ca^3b^2$$

By adding $(2); (3); (4)$ we obtain (1).
Equality holds for $a = b = c$. $\qquad\qquad\square$

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