PP44489

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$OI + OI_a + OI_b + OI_c \leq 4R\sqrt{3}$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$OI + OI_a + OI_b + OI_c \overset{\text{CBS}}{\leq}$$
$$\leq \sqrt{(1^2 + 1^2 + 1^2 + 1^2)(OI^2 + OI_a^2 + OI_b^2 + OI_c^2)} =$$
$$= \sqrt{4(R^2 - 2Rr + R^2 + 2Rr_a + R^2 + 2Rr_b + R^2 + 2Rr_c)} =$$
$$= 2\sqrt{4R^2 - 2Rr + 2R(r_a + r_b + r_c)} =$$
$$= 2\sqrt{4R^2 - 2Rr + 2R(4R + r)} =$$
$$= 2\sqrt{4R^2 - 2Rr + 8R^2 + 2Rr} =$$
$$= 2\sqrt{12R^2} = 2 \cdot 2\sqrt{3} \cdot R = 4\sqrt{3}R$$

Equality holds for $a = b = c$. □

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