PP44508

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc} \frac{w_a^2}{(s-b)(s-c)} \leq \frac{s^2-2r^2-8Rr}{r^2}$$

Solution by Daniel Sitaru, Claudia Nănuţi.

$$\sum_{cyc} \frac{w_a^2}{(s-b)(s-c)} = \sum_{cyc} \frac{\frac{4}{(b+c)^2} \cdot bcs(s-a)}{(s-b)(s-c)} \leq$$

$$\overset{\text{AM-GM}}{\leq} \sum_{cyc} \frac{\frac{4}{4bc} \cdot bcs(s-a)}{(s-b)(s-c)} = s \sum_{cyc} \frac{s-a}{(s-b)(s-c)} =$$

$$= \frac{s}{(s-a)(s-b)(s-c)} \cdot \sum_{cyc} (s-a)^2 =$$

$$= \frac{s^2}{s(s-a)(s-b)(s-c)} \cdot \sum_{cyc} (s^2 - 2sa + a^2) =$$

$$= \frac{s^2}{F^2}(3s^2 - 2s \cdot 2s + 2s^2 - 2r^2 - 8Rr) =$$

$$= \frac{s^2}{r^2 s^2}(s^2 - 2r^2 - 8Rr) = \frac{s^2-2r^2-8Rr}{r^2}$$

Equality holds for $a = b = c$.

□

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