PP44539

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\prod_{cyc} \cos\frac{A-B}{2} \leq \frac{R+2r}{2R}$$

Solution by Daniel Sitaru and Claudia Nănuţi.
It is known that:

$$\prod_{cyc} \cos\frac{A-B}{2} = \frac{s^2+r^2+2Rr}{8R^2}$$

Remains to prove that:

$$\frac{s^2+r^2+2Rr}{8R^2} \leq \frac{R+2r}{2R}$$
$$s^2+r^2+2Rr \leq 4R(R+2r)$$
$$s^2 \leq 4R^2+8Rr-r^2-2Rr$$
$$s^2 \leq 4R^2+6Rr-r^2 \text{ (to prove)}$$
$$s^2 \overset{\text{GERRETSEN}}{\leq} 4R^2+4Rr+3r^2 \leq 4R^2+6Rr-r^2$$
$$3r^2 \leq 2Rr-r^2$$
$$2Rr \geq 4r^2$$
$$R \geq 2r \text{ (Euler)}$$

Equality holds for: $a=b=c$.

□

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