PROPOSED PROBLEM

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$Rh_ah_bh_c \geq 2rw_aw_bw_c$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$Rh_ah_bh_c \geq 2rw_aw_bw_c$$

$$R \cdot \frac{8F^3}{abc} \geq 2r \cdot \frac{8a^2b^2c^2}{(a+b)(b+c)(c+a)} \cdot \prod_{cyc} \cos\frac{A}{2}$$

$$\frac{8RF^3}{4RF} \geq \frac{16r \cdot (4RF)^2}{(a+b)(b+c)(c+a)} \cdot \frac{s}{4R}$$

$$2F^2 \geq \frac{16r \cdot 16R^2F^2}{2s(s^2+r^2+4Rr)} \cdot \frac{s}{4R}$$

$$1 \geq \frac{16 \cdot 8R^2r}{2(s^2+r^2+2Rr)} \cdot \frac{1}{4R}$$

$$1 \geq \frac{16Rr}{s^2+r^2+2Rr} \Leftrightarrow s^2+r^2+2Rr \geq 16Rr \Leftrightarrow$$

$$\Leftrightarrow s^2 \geq 14Rr - r^2 \text{ (to prove)}$$

$$s^2 \overset{\text{GERRETSEN}}{\geq} 16Rr - 5r^2 \geq 14Rr - r^2 \Leftrightarrow$$

$$\Leftrightarrow 2Rr \geq 4r^2$$

$$R \geq 2r \text{ (Euler)}$$

Equality holds for $a = b = c$.

□

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