PP44600

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\sum_{cyc}\frac{\cos A}{\sin A}\geq\frac{s}{3r}$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$\sum_{cyc}\frac{\cos A}{\sin A}=\sum_{cyc}\frac{b^2+c^2-a^2}{2bc\sin A}=\sum_{cyc}\frac{b^2+c^2-a^2}{4F}=$$

$$=\frac{1}{4F}\left(\sum_{cyc}b^2+\sum_{cyc}c^2-\sum_{cyc}a^2\right)=\frac{1}{4F}\sum_{cyc}a^2=$$

$$=\frac{1}{4rs}\cdot 2(s^2-r^2-4Rr)=\frac{s^2-r^2-4Rr}{2rs}\geq\frac{s}{3r}\Leftrightarrow$$

$$\Leftrightarrow 3(s^2-r^2-4Rr)\geq 2s^2$$

$$3s^2-3r^2-12Rr\geq 2s^2$$

$$s^2\geq 12Rr+3r^2 \text{ (to prove)}$$

$$s^2\overset{\text{GERRETSEN}}{\geq}16Rr-5r^2\geq 12Rr+3r^2$$

$$4Rr\geq 8r^2$$

$$R\geq 2r \text{ (Euler)}$$

Equality holds for $a=b=c$.

□

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