PP44603

MIHÁLY BENCZE - ROMANIA

In all triangles ABC holds:

$$\frac{s^2-3r^2}{8Rr}+\frac{17r}{18R}\geq\frac{71}{36}$$

Solution by Daniel Sitaru and Claudia Nănuţi.

$$\frac{s^2-3r^2}{8Rr}+\frac{17r}{18R}\geq\frac{71}{36}$$

$$\frac{s^2-3r^2}{8Rr}\geq\frac{71}{36}-\frac{17r}{18R}$$

$$\frac{s^2-3r^2}{8Rr}\geq\frac{71R-34r}{36R}$$

$$\frac{s^2-3r^2}{2r}\geq\frac{71R-34r}{9}$$

$$9s^2-27r^2\geq 142Rr-68r^2$$

$$9s^2\geq 142Rr-41r^2 \text{ (to prove)}$$

$$9s^2\overset{\text{GERRETSEN}}{\geq}9(16Rr-5r^2)\geq 142Rr-41r^2$$

$$144Rr-45r^2\geq 142Rr-41r^2$$

$$2Rr\geq 4r^2$$

$$R\geq 2r \text{ (Euler)}$$

Equality holds for $a=b=c$.

□

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